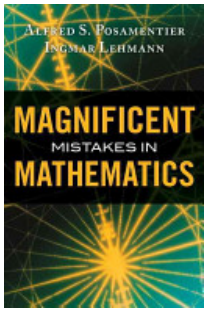
Magnificent Mistakes in Mathematics, 2013, Prometheus Books, ISBN 978-1-61614-747-1 (hbk), 198 pp. by *Alfred S. Posamentier and Ingmar Lehmann*



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We all learn from our mistakes (and not only the mathematical ones). This is one of the reasons these authors wrote the book. By showing what kind of mistakes can be made in mathematics, and to what absurd conclusions that may lead, it is hoped that the reader understands better the rules of the game and be more careful in jumping to conclusions.

However, we may learn not only from our own mistakes. There have been many historical mistakes, made by leading mathematicians. The first chapter is a collection of such examples. Pythagoras was mistaken when he thought that nature could be completely explained with natural numbers and their ratios. There have been historical mistakes in the calculation of π, and many wrong attempts have been made to prove Fermat's last theorem, Goldbach's conjecture, or solve the 4 colour problem, and many other such famous problems. Galileo, Euler, Fermat, Legendre, Poincaré, Einstein, they all made mistakes and often in published papers. Gauss seems to be a glorious exception to this rule. No errors are known in his published papers. This chapter is an enumeration of summaries of these historical errors, although a complete book could be devoted to each of them, why and how the wrong conclusion was made and what kind of research this has started. For example, a wrong calculation of a notorious gambler *Chevalier de Méré* caused him to loose. He asked Pascal to explain what seemed to him a paradox, and the correspondence between Pascal and Fermat can be considered to be the start of probability theory. And we all know that the attempts to prove Fermat's last theorem has given rise to a many new mathematical results.

The subsequent chapters discuss arithmetical, algebraic, geometrical and statistical mistakes. Here we find many obvious errors that are commonly committed by students like division by zero, or violating the rules of distributivity ($\sqrt{a+b} = \sqrt{a}+\sqrt{b}$ or $\frac{a}{b}+\frac{c}{d} = \frac{a+c}{b+d}$ and the likes). Also jumping too soon to a general conclusion is common. Infinite sums, and working with ∞ often leads to false results. Of a somewhat different nature are the rounding errors of digital calculators or computers that may play dirty tricks on us. Sometimes wrong logic may lead to correct answers. Arguments are then needed to convince the student of the low score. All these are familiar and we are desperate when students persist sinning against them. Many examples of mistakes can however be reduced to the same error made under a slightly disguised form. So there is basically a lot of repetition which makes these chapters a bit dull from time to time. There are however also errors that are counter intuitive or that have some pitfalls and that are often used in quizzes or to astonish the innocent reader with an apparent paradox. For example suppose the earth is a perfect sphere. Put a rope around the equator and enlarge it by 1 meter. Then keep this longer rope at an equal distance above the surface. Can a mouse pass under the rope? Our intuition says no. However, computation results in a different result. The cat can pursue the mouse at the other hemisphere as well. There are the optical illusions and impossible figures in a geometrical context. A famous geometrical puzzle showing that 65 = 64 popularized by Lewis Carroll is shown in the figure on the right. It has happened that mistakes were deliberately introduced as a prank. Martin Gardner presented in his April 1975 column of *Scientific American* a map that would require 5 colours, which turned out to be an April fools joke on his readers. Stories like this and the more recreational pitfalls will keep you reading to the end. A not always very deep, but a broad and diverse collection of examples. *Adhemar Bultheel*